Do the following series converge or diverge?

a) $\displaystyle\sum_{n=0}^{\infty} \frac{5n+2}{n^3+1}$

b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1+\sqrt{5}}{2}\right)^n$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

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a) $\sum_{n=0}^{\infty} \frac{5n+2}{n^3+1}$

Let $f(n) = \frac{5n}{n^3} = \frac{5}{n^2}$.

$\int x$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{5/n^2}{5n+2/n^3+1}$$

$$= \lim_{n\to\infty} \frac{5n^3+5}{5n^3+2n^2}$$

$$= \lim_{n\to\infty} \frac{1+\frac{1}{n^3}}{1+\frac{2}{5n}}$$

$$= 1$$

$\sum_{n=1}^{\infty} \frac{5}{n^2}$ converges, $\therefore \sum_{n=0}^{\infty} \frac{5n+2}{n^3+1} = 2 + \sum_{n=1}^{\infty} \frac{5n+2}{n^3+1}$ converges.

b) $\sum_{n=1}^{\infty}\left(\frac{1+\sqrt{5}}{2}\right)^n$ is made up of exponential functions x^n

and $x=\frac{1+\sqrt{5}}{2}>1$. Hence, $\sum_{n=1}^{\infty}\left(\frac{1+\sqrt{5}}{2}\right)^n$ diverges.

c) $\sum_{n=1}^{\infty}\frac{\ln n}{n^2}$

$$f(n)=\frac{\ln n}{n^2}$$

$$f'(n)=\frac{\frac{1}{n}n^2-\ln n\cdot 2n}{n^4}$$

$$=\frac{n-2n\ln n}{n^4}$$

$$=\frac{1-2\ln n}{n^4}$$

$\therefore f'(n)<0 \Rightarrow f(n)$ is decreasing when

$n>e^{1/2}=1.649$

$\Rightarrow f(n)$ is decreasing for $n\geq 2$

$$\int_2^{\infty}\frac{\ln n}{n^2}\,dn$$

$$=\ln n\left(-\frac{1}{n}\right)\Big|_2^{\infty}-\int_2^{\infty}\frac{1}{n}\left(-\frac{1}{n}\right)dn$$

$$=0-\left(-\frac{\ln 2}{2}\right)+\left(-\frac{1}{n}\right)\Big|_2^{\infty}$$

$$=\frac{\ln 2}{2}-\left(0-\frac{1}{2}\right)$$

$$=\frac{\ln 2+1}{2}$$

$f(n)$ is decreasing and $f(n)>0$ from $n=2$ to infinity.

$\int_2^{\infty}\frac{\ln n}{n^2}\,dn$ converges $\Rightarrow \sum_{n=2}^{\infty}\frac{\ln n}{n^2}$ converges.

$\therefore \sum_{n=1}^{\infty}\frac{\ln n}{n^2}=\ln 1+\sum_{n=2}^{\infty}\frac{\ln n}{n^2}$ also converges.